

If $y = Bx$, where B is known exactly and x has an associated error, σ_x , then the error on y, σ_y is given as

$$\begin{aligned}\sigma_y &= \sqrt{\left(\frac{dy}{dx}\right)^2 \sigma_x^2} \\ \frac{dy}{dx} &= \frac{d}{dx} Bx \rightarrow B \\ \sigma_y &= \sqrt{B^2 \sigma_x^2} \\ \sigma_y &= B\sigma_x\end{aligned}$$

For multiple variables that have independent error estimates, then you add in quadrature. So, if now $y = Ax + Bz$, where A and B are known exactly but x and z have errors, σ_x and σ_z , respectively, then

$$\sigma_y = \sqrt{\left(\frac{\delta y}{\delta x}\right)^2 \sigma_x^2 + \left(\frac{\delta y}{\delta z}\right)^2 \sigma_z^2} \quad (4)$$

Note that the equation is now using partial derivatives (δ) instead of full derivatives because y is a function of both x and z ($y(x,z)$). To write it out explicitly,

$$\begin{aligned}\frac{\delta y}{\delta x} &= \frac{\delta}{\delta x} (Ax + Bz) \rightarrow A \\ \frac{\delta y}{\delta z} &= \frac{\delta}{\delta z} (Ax + Bz) \rightarrow B \\ \sigma_y &= \sqrt{(A\sigma_x)^2 + (B\sigma_z)^2}\end{aligned}$$

In another example, let $y = xz$ where both x and z have independent errors. Following from Equation (4),

$$\begin{aligned}\sigma_y &= \sqrt{\left(\frac{\delta}{\delta x}(xz)\right)^2 \sigma_x^2 + \left(\frac{\delta}{\delta z}(xz)\right)^2 \sigma_z^2} \\ \frac{\delta}{\delta x}(xz) &\rightarrow z \\ \frac{\delta}{\delta z}(xz) &\rightarrow x \\ \sigma_y &= \sqrt{(z\sigma_x)^2 + (x\sigma_z)^2}\end{aligned}$$

As long as the errors are independent on each other, then you can keep adding terms to Equation (4). Usually, errors are independent.